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On simulation of optimal strategies and Nash equilibrium in the financial market context

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Abstract Unlike physical time series, stock market prices may be affected by the predictions made by market participants with conflicting interests. This is the domain of game theory. Therefore, we propose a Stock Exchange Game Model (SEGM) to model this phenomenon. In SEGM, player strategies are to set their buying and selling levels for the next iteration via the autoregressive model AR(p) of order p selected by minimizing deviations from Nash Equilibrium (NE). NE represents the assumption of optimal behavior by market participants. The objective of SEGM is to simulate financial and other time series that are affected by predictions of the participants and to test the assumption of optimal player behavior, using a 'virtual' stock exchange. The simulation of SEGM suggests that NE is close to the Wiener model. This is a new explanation of the Random Walk (RW) model of the efficient market theory. To compare the simulation results with real data, the efficient market hypothesis was also tested, using financial time series of eight assets. The SEGM software is implemented in Java applets and can be run using a browser with Java support. The main web site is in http://soften.ktu.lt/~mockus.

Keywords Optimization · Nash equilibrium · Stock exchange · Time series · Wiener model

1 Introduction

Financial markets experienced extraordinary events in 2007 and 2008. See [2,6] for in-depth discussions. Simulation of such complex and diverse processes may be too difficult. Therefore, it is of interest to create simple models that explain at least some of the interesting properties of financial and related markets.

A common feature of all these mechanisms is that future financial data depend on predictions. Another common feature is balancing of conflicting interests. The game-theoretical

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concept for that is Nash Equilibrium (NE) [16]. In simple terms NE can be defined as a contract of players that provides no incentives to brake it. This means that no player can get additional profit by breaking NE. In this paper, a simple model reflecting both these features is proposed and investigated. We refer to it as a Stock Exchange Game Model (SEGM).

Recently financial markets have became a more accessible investment tool, not only for professional investors, but also for much wider group of people. Consequently, the markets are not only related to macroeconomic parameters, but also they influence everyday life in a more direct way. Therefore, they constitute a mechanism that has important and direct social impacts. The characteristic that all financial markets have in common is uncertainty related to their short-term and long-term future state. This feature is undesirable for investors, but also is unavoidable whenever the market is selected as an investment tool.

One of the strategies to cope with this uncertainty is to try to reduce it. Predicting financial markets is one of the instruments in this process. Different individuals may regard the problem differently. Therefore, providing research tools to interested parties, such as graduate students in relevant fields, may contribute to further understanding of these markets. To achieve that, the SEGM is implemented as a Java applet, published in an open web-site and can be run by any browser with Java support: http://pilis.if.ktu.lt/~jmockus/stockedvinas/indre.html.

The market prediction task divides researchers into two groups: those who believe that we can devise mechanisms for predicting the market, and those who believe that the market is efficient and, whenever any new information appears, the market absorbs it by correcting itself. A convenient mathematical tool representing the latter model is the Wiener process, which implies that the best possible prediction of the next value is based only on the present value [8,17]. The Wiener model is often called the Random Walk (RW).

In this paper, a non-traditional game-theoretical justification of the RW is proposed as a working hypothesis. The stock exchange is regarded as a game of competing players trying to maximize the wins by selling stocks[1] at high prices and buying them at lower prices. Selling/buying levels are based on next-day[2] predictions by autoregressive models $AR(p)$ of order p. In real life, the asset holders predict using their own specific models. We consider the $AR(p)$ model as a reasonable first approximation of such forecasting strategies. We assume that in SEGM players control the order p of the autoregressive model $AR(p)$. Players make profit if predict correctly. The experimental calculations suggest that under these assumptions NE is achieved if all major players predict stock prices using the Wiener model ($p = 1$).

Below there is a short list of traditional methods to predict time series. Three major groups are considered: technical, statistical, and fundamental analysis.

i Technical analysis.
The idea behind the technical analysis is that stock prices move in trends dictated by the constantly changing attributes of investors in response to different forces. Using technical data such as price, volume, the highest and lowest prices per trading period, a technical analyst uses charts to predict future stock movements. Price charts are used to detect trends, these trends are assumed to be based on supply and demand issues which often have cyclic or noticeable patterns. Technical analysts, also known as chartists, attempt to predict the market by tracing patterns that come from the study of charts which describe historical data of the market.

[1] The term stock means an asset of financial market, for short.

[2] The term 'next-day' means next time unit, for short.

ii Statistical analysis.
The examples are the Auto Regression Moving Average (ARMA) model for short-time prediction and the Fractional Integration (ARFIMA) model for long-time predictions. Theoretical and practical results are in [9,21]. The optimization approach to ARFIMA is described in [15]. Comparison of the Artificial Neural Network (ANN) and ARMA models is given in [18]. See [3] for a survey of fractional integration methods and applications in economics and finance.
The Autoregressive Conditional Heteroskedastic processes are also applied to model financial data [5,7]. The frequently used model is the GARCH(p,q) sequence [10,11].

iii Fundamental analysis.
Fundamental analysts study the intrinsic value of a stock and they invest in it if they estimate that its current value is lower than its intrinsic value. The analysts that use this method of prediction use fundamental data in order to have a clear picture of the firm (industry or market) they will choose to invest in. They compute the ‘real’ value of the asset they will invest in and they determine this value by studying variables such as the growth, dividend pay-out, interest rates, risk of investment, sales level, tax rates, and so on. Their aim is to calculate the intrinsic value of an asset. To this end, they apply a simple trading rule. If the intrinsic value of the asset is higher than the value it holds in the market, they invest in it. If not, they consider it a bad investment and avoid it. Some empirical test of chartists versus fundamentalist hypothesis is presented in [4].

It is unlikely that the SEGM provides a complete description of the behavior of all stock market participants. For example, ordinary stockholders[3] are not trying to achieve equilibrium, they are just looking for a greater profit. However, the model may be useful in explaining prediction peculiarities of financial time series, where the prices are affected by predictions of market participants. The model represents, in a simple game-theoretical form, basic arguments of efficient market theory and, therefore, is a natural initial approximation. Thus, RW can be regarded as a benchmark of minimal efficiency convenient for evaluating more sophisticated prediction techniques. The objective of this paper is to provide a web-based tool for implementation and investigation of the NE model of financial markets. We regard the results and tools, described in this paper, as an example encouraging further research along similar lines by interested persons, not just professional investors.

To compare the simulation results with real data, the efficient market hypothesis was also tested, using financial time series of eight assets. To reflect risk-neutral users, the Autoregressive by Absolute Values AR - ABS(p) model of order p was used. The parameter p was optimized for each asset. The RW ($p = 1$) provided best results for cocoa futures prices and for the simulated Wiener process. In other cases results of RW were not optimal but close.

2 Stock exchange game model (SEGM)

The purpose of Stock Exchange Game Model (SEGM) is to explain the relationship between the RW and Nash equilibrium and to investigate what other results can be obtained using this simple model. Stock price prediction models are based on the analysis of the past data in the form of time series, as usual. In this paper, a different approach is investigated. The scientific objective of this approach is to test the hypothesis, that stock exchange can be approximately described as a game of players using strategies, based on the Nash equilibrium. Stock prices

[3] The term stockholder means a holder of financial asset, for short.

are primarily the result of the game of major stockholders with smaller random deviations, representing large numbers of small investors. Investment decisions depend on stockholders' predictions of the future stock prices and expected dividends. This model is used to investigate what could be learned about the basics of market theory based on the simple game-theoretical stock exchange model.

SEGM assumes that each player predicts stock prices by the autoregressive model $AR(p)$ of order p. Scale parameters a of the $AR(p)$ model (11) are estimated using the standard least squares algorithm for different p. Actual stockholders use their own ways for predicting. We regard the $AR(p)$ model as the simplest initial approximation of the prediction processes.

We start a formal description by considering the simple case of I major players $i = 1, \dots, I$ and a single joint-stock company. The following notation is used:

$z(t, i)$ is the price at time t, predicted by the player i,
$Z(t)$ is the actual[4] price at time t,
$P(t, i)$ is the actual profit accumulated at time t by player i,
$\delta(t)$ are dividends at time t,
$\alpha(t)$ is the yield at time t,
$\beta(t, i)$ is the relative stock price change as predicted by the player i at time t,
$\beta(t, i) = (z(t + 1, i) - Z(t))/Z(t)$,
$k_b(i) > 0$, is a relative buying level, of the player i,
$k_s(i) < 0$ is a relative selling level of the player i,
$z(t, i)$ is the price at time t predicted by the the player i,
$\varepsilon(t)$ is the noise (Gaussian random number with zero expectation and variance reflecting stock volatility),
$p(t, i) = \beta(t, i) - \alpha + \delta$ is a relative profit predicted by the player i at time t.

The assumed strategy is that at time t the player i buys the number N of stocks, if $p(t, i) \geq k_b(i) > 0$, and sells N stocks, if $p(t, i) \leq k_s(i) < 0$.

The buying price level of the player i at time t is

$$z_b(t, i) = z(t + 1, i)/(1 - \delta(t) + \alpha(t) + k_b(i)). \quad (1)$$

The selling price level of the player i at time t is

$$z_s(t, i) = z(t + 1, i)/(1 - \delta(t) + \alpha(t) + k_s(i)), \quad (2)$$

where $z(t + 1, i)$ is the stock price at time $t + 1$ as predicted by investor i.

In terms of the price levels, the buying-selling strategy $S(t, i)$

$$S(t, i) = \begin{cases} \text{buy } N \text{ stocks}, & \text{if } Z(t) \leq z_b(t, i), \\ \text{sell } N \text{ stocks}, & \text{if } Z(t) \geq z_s(t, i), \\ \text{wait}, & \text{if } z_b(t, i) < Z(t) < z_s(t, i). \end{cases} \quad (3)$$

The market buying price at time t is the largest buying price of players $i = 1, \dots, I$

$$z_b(t) = z_b(t, i^{\max}).$$

where

$$i^{\max} = \arg\max_i z_b(t, i).$$

[4] The term 'actual' means simulated by SEGM.

The market selling price at time t is the lowest selling price of players $i = 1, \ldots, I$

$$z_s(t) = z_b(t, i^{\min}),$$
$$i^{\min} = \arg\min_i z_s(t, i).$$

The actual price of a stock at time $t + 1$ is defined as the price of a previous deal of major stockholders plus the noise representing the remaining small stockholders.

$$Z(t+1) = \begin{cases} z_b(t) + \varepsilon(t+1), & \text{if } p(t, i^{\max}) \geq k_b(i^{\max}), \\ z_s(t) + \varepsilon(t+1), & \text{if } p(t, i^{\min}) \leq k_s(i^{\min}), \\ Z_t + \varepsilon(t+1), & \text{otherwise.} \end{cases} \quad (4)$$

The number of stocks owned by the player i at time $t + 1$ is

$$N(t+1, i) = N(t, i) + N, \quad \text{if } p(t, i) \geq k_b(i), \quad (5)$$
$$N(t+1, i) = N(t, i) - N, \quad \text{if } p(t, i) \leq k_s(i), \quad (6)$$
$$N(t+1, i)) = N(t, i), \quad \text{otherwise.} \quad (7)$$

Here N is the number of stocks allocated for buying-selling operations. A consumer gets profit as the difference between selling and buying prices, if his/her predictions are correct.

The profit of the player i accumulated at time T is

$$P(T, i) = N(T, i)Z(T) - N(0, i)Z(0) - \sum_{t=0}^{T}((N(t+1, i) - N(t, i))Z(t). \quad (8)$$

We were considering a single level strategy so far. The single level strategy is easy to explain and convenient to illustrate. Figures 1 and 2 illustrate the single level strategy for the 5% relative level.

However, according to the expert opinion of professional brokers we have interviewed, we need at least three buying-selling price levels $l = 1, 2, 3$ to explain the behavior of major stockholders. The level $l = 1$ means to buy-sell a minimal number of stocks. The level $l = 3$

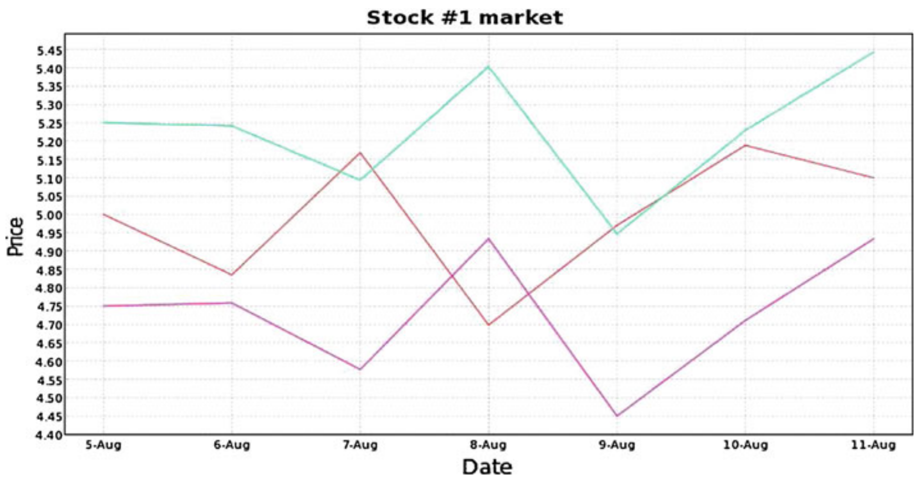

Fig. 1 Stock price and single buying-selling levels of Wiener model

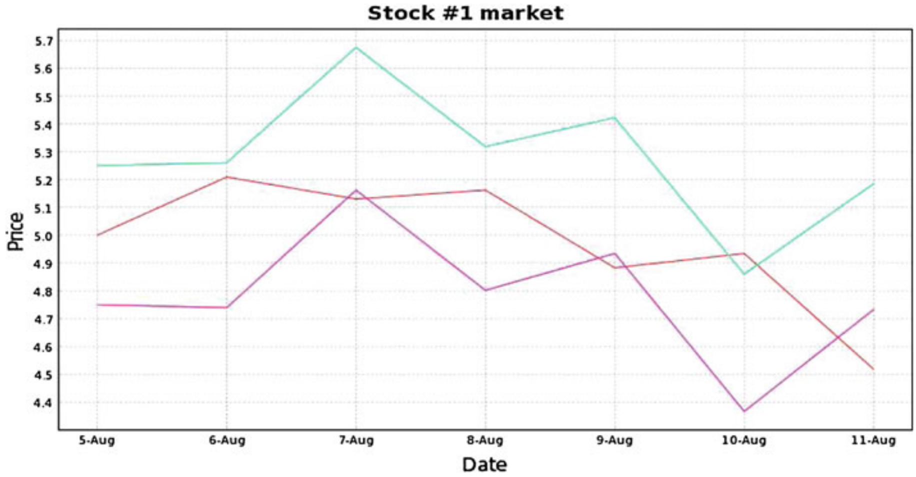

Fig. 2 Stock price and single buying-selling levels of $AR(p)$, $p = 9$ model

means to buy-sell as much stocks as possible, and the level $l = 2$ means to buy-sell a part of the maximum level. Below is a formal description of three-level strategy.

The number $k_b(i, l)$ denotes the relative buying level l, of the player i

$$k_b(i, l) > 0, \ k_b(i, l) < k_b(i, l - 1)$$

A player is buying:

- maximal amount $N = N_3$, if $l = 3$,
- average amount $N = N_2$, if $l = 2$,
- minimal amount $N = N_1$ if $l = 1$.

The number $k_s(i, l)$ is the relative selling level l of the player i,

$$k_s(i, l) < 0, \ k_s(i, l) > k_s(i, l - 1)$$

A player is selling:

- maximal amount $N = N_3$, if $l = 3$,
- average amount $N = N_2$, if $l = 2$,
- minimal amount $N = N_1$, if $l = 1$.

The buying-selling prices of the player i at time t depend on the levels l

$$\begin{aligned} z_b(t, i, l) &= z(t + 1, i)/(1 - \delta(t) + \alpha(t) + k_b(i, l)) \\ z_s(t, i, l) &= z(t + 1, i)/(1 - \delta(t) + \alpha(t) + k_s(i, l)). \end{aligned} \qquad (9)$$

The market buying price is defined as the highest buying price

$$z_b(t) = z_b(t, i^{\max}, l^{\max}),$$

where

$$(i^{\max}, l^{\max}) = \arg\max_{i,l} z_b(t, i, l).$$

The market selling price is defied as the lowest selling price

$$z_s(t) = z_s(t, i^{\min}, l^{\min}),$$

where

$$(i^{\min}, l^{\min}) = \arg\min_{i,l} z_s(t, i, l).$$

The actual price of a stock at time $t + 1$ is defined as the price of a previous deal of major stockholders plus the noise representing the remaining small stockholders. The number of stocks owned by the player i at time $t + 1$ is as follows:

$$\begin{aligned}
&N(t+1,i) = N(t,i) + N_3, && \text{if } p(t,i,3) \geq k_b(i,3),\\
&N(t+1,i) = N(t,i) - N_3, && \text{if } p(t,i,3) \leq k_s(i,3),\\
&N(t+1,i) = N(t,i) + N_2, && \text{if } p(t,i,2) \geq k_b(i,2),\ p(t,i,3) < k_b(i,3),\\
&N(t+1,i) = N(t,i) - N_2, && \text{if } p(t,i,2) \leq k_s(i,2),\ p(t,i,3) > k_s(i,3),\\
&N(t+1,i) = N(t,i) + N_1, && \text{if } p(t,i,1) \geq k_b(i,1),\ p(t,i,2) < k_b(i,2),\\
& && p(t,i,3) < k_b(i,3),\\
&N(t+1,i) = N(t,i) - N_1, && \text{if } p(t,i,1) \leq k_s(i,1),\ p(t,i,2) > k_s(i,2),\\
& && p(t,i,3) > k_s(i,3),\\
&N(t+1,i)) = N(t,i), && \text{otherwise.}
\end{aligned} \tag{10}$$

The profit of the player i accumulated at time T is defined by expression (8).

2.1 Definition of prediction parameters

The profit (8) of the player i depends on the accuracy of prediction $\beta(t + 1, i)$ and random numbers $\varepsilon(t, i)$.

Assume that the player i predicts using the $AR(p)$ model of order p with the parameter $p = x_i$. Then the predicted price

$$z(t+1,i) = \sum_{k=1}^{x_i} a_i^k \, z(t-k,i) + \varepsilon(t+1,i). \tag{11}$$

Here the scale parameters $a_i = (a_i^k, k = 1, \ldots, x_i)$ are defined by the condition

$$a_i = \arg\min_{a_i} \sum_{s=1}^{t} \varepsilon_i^2(s,i), \tag{12}$$

where

$$\varepsilon(s,i) = z(s) - \sum_{k=1}^{x_i} a_i^k \, z(s-k,i). \tag{13}$$

Note, that SEGM is for stock exchange simulation by generating time series of virtual stock prices and is not intended for actual predictions.

2.2 Search for Nash equilibrium

Suppose that $x_i = 0, 1, \ldots, L$. Here $x_i = 0$ refers to the Wiener model with the parameters $p = 1, a_1 = 1$, and $x_i = l > 0$ means the $AR(p)$ model with the parameter $p = l$.

Denote by $x^0 = (x_i^0, i = 1, \ldots, I)$ a 'contract' vector and by $x^1 = (x_i^1,\ i = 1, \ldots, I)$ a 'non-contract' vector, where

$$x_i^1 = \arg\max_{x_1} P(T, i, x_i, \bar{x}_i^0), \quad i = 1, \ldots, I. \tag{14}$$

Here $\bar{x}_i^0 = (x_k^0,\ k = 1, \ldots, I, k \neq i)$, and $P(T, i, x_i, \bar{x}_i^0)$ is the accumulated profit of the player i predicted using the $AR(p)$ model with $p = x_i$ while other players $k \neq i$ are predicting by the $AR(p)$ model with $p = x_k^0$.

We search for such a contract vector $x^* = (x_i^*,\ i = 1, \ldots, I)$ that minimizes the sum of differences between 'non-contract' and 'contract' $x^0 = (x_i^0, i = 1, \ldots, I)$ profits

$$x^* = \arg\min_{x^0} \sum_{i=1}^{I} ((P(T, i, x_i^1, \bar{x}_i^0) - P(T, i, x_i^0, \bar{x}_i^0)). \tag{15}$$

Expression (15) means the minimum of the sum of players' profits obtained by violating the NE strategy while other players follow NE strategies. Therefore, the contract vector x^* defines the Nash equilibrium exactly if the sum (15) is zero. The contract vector x^* defines the Nash equilibrium approximately if the sum is less than the simulation error.

Expression (15) defines a stochastic optimization problem on a set of L^I integer vectors. The Bayesian Heuristic Approach [14] is for this problem. Other well-known global optimization methods [19,20] can be applied, too. However, the convergence rate is slow. Therefore, in the following examples, an approximate equilibrium was obtained by comparing average profits of the Wiener and $AR(p)$ models. In simulations the average $AR(p)$ profits did not exceed the profits of the Wiener model. This indicates that, in this setup, the Wiener model approximately satisfies the NE conditions.

3 Examples of the stock exchange game model

Figure 1 shows three lines in a single level model. The top and bottom lines represent 5% buying and selling levels $z_b(t, i),\ z_s(t, i)$ of the player $i = 1$ in six days, obtained by the Wiener model. The middle line shows the simulated stock price $Z(t)$. According to the Wiener model, the buying-selling levels are parallel to the stock price line with a delay of one day. Figure 2 shows similar results, obtained using the autoregressive model $AR(p),\ p = 9$. Here the buying-selling levels are not parallel to the price line.

In Fig. 3, the lower line represents the profit of a player using the $AR(p),\ p = 9$ model. The upper line shows profits of the three remaining players using the Wiener model. Since all these players are using the same buying-selling levels, all the three lines merge into a single upper line. The figure illustrates that in SEGM the profit is not improved using the $AR(p),\ p = 9$ comparing with the Wiener model. This suggests that in this case the RW model approximately provides Nash equilibrium.

A way of testing the RW model is to investigate the reaction to well known deliberately non-NE strategies of some major stockholder. An example is George Soros currency manipulation by selling a large amount of assets to buy back these and other assets later at an artificially lowered price. An important feature of this scenario is that the player 'Soros'

Fig. 3 Profit of $AR(p)$, $p = 9$ versus Wiener model

Fig. 4 Profits in ‘Soros-vs-AR(p, $p = 9$)’ game

makes great profit if other players are predicting prices by the previous data, for example, by $AR(p)$, $p > 1$. In Fig. 4, the lower line represents the profits of three players using the $AR(p)$, $p = 9$ model. The upper line shows the profit of a player using the ‘Soros’ model. Note that this profit is almost 1,000 greater, thus, the profits of $AR(p)$, $p = 9$ players appear like small constant values. In Fig. 5, the middle line represents the stock price in the ‘Soros-vs-AR(p, $p = 9$)’ game. The other six lines show buying and selling levels.

Fig. 5 Stock price in 'Soros-vs-AR(p), $p = 9$' game

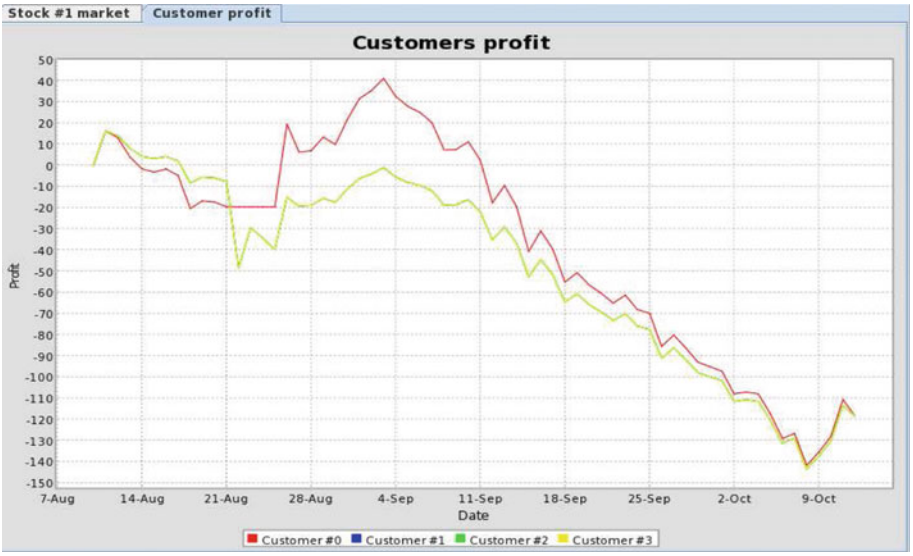

Fig. 6 Profits in 'Soros-vs-Wiener' game

Figure 6 shows that player Soros makes no profit if the remaining players use the Wiener model. In this figure, the lower line represents the profits of three players using the Wiener model. The upper line shows profit of the player using the 'Soros' model. Here 'Soros' obtains no profit at the end of the game.

4 Autoregressive by absolute deviation model ($AR - ABS(p)$)

To compare the simulation results with real data, an efficient market hypothesis has also been tested, using financial time series of eight assets. Details are in the next section. To reflect risk-neutral market participants, the Autoregressive by Absolute Values $AR-ABS(p)$ model of order p was used. The order p was optimized for each asset.

The method of least squares is sensitive to large deviations [1]. Therefore, the replacement of squares by absolute values is beneficial, if the customers' utility function is linear. The linear utility function represents risk-neutral behavior.

Define the $AR - ABS(p)$ model of order p as

$$w_t = \sum_{i=1}^{p} a_i w_{t-i} + \varepsilon_t. \tag{16}$$

We assume that

$$w_{t-i} = 0, \ \varepsilon_{t-i} = 0, \quad \text{if} \quad t \leq i. \tag{17}$$

Here w_t is the next-day prediction and w_{t-1} is the value observed today, w_{t-i-1} is the value observed i days ago, ε_t are independent Gaussian $(0, \sigma)$ random numbers, where σ is the standard deviation and a_i are scale parameters.

Using equalities (16), we define residuals by the expressions

$$\begin{aligned} \varepsilon_1 &= w_1, \\ \varepsilon_2 &= w_2 - a_1 w_1, \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \varepsilon_t &= w_t - a_1 w_{t-1} - \cdots - a_p w_{t-p}. \end{aligned} \tag{18}$$

Scale parameters $a = (a_i, \ i = 1, \ldots, p)$ are defined by the condition

$$a = \arg\min_a \sum_{t=1}^{T} |\varepsilon_t| \tag{19}$$

This explains the acronym $AR - ABS(p)$ meaning that we minimize the sum of absolute values of the residuals.

4.1 Minimization of residuals in $AR - ABS(p)$ models

To minimize $f(x)$, we apply linear programming.

$$\min_{a,u} \sum_{t=1}^{T} u_t \tag{20}$$

$$u_t \geq \varepsilon_t, \quad t = 1, \ldots, T \tag{21}$$

$$u_t \geq -\varepsilon_t \quad t = 1, \ldots, T \tag{22}$$

$$u_t \geq 0, \ \varepsilon_t \quad t = 1, \ldots, T \tag{23}$$

Here
$u = (u_t^1, u_t^2, \ t = 1, \ldots, T), a = (a_i^1, a_i^2, \ i = 1, \ldots, p)$,
$u_t = u_t^1 - u_t^2, \ t = 1, \ldots, T), a_i = a_i^1 - a_i^2, \ i = 1, \ldots, p$,
$u_t^k \geq 0, \ a_i^k \geq 0, \ k = 1, 2.$

5 Examples of financial time series

Time series are selected to represent various asset sectors. Nine assets are used in evaluations. The prices are predicted. Eight assets are real-world futures close prices. The ninth 'asset' is a random simulation. Random independent Gaussian numbers are summed up cumulatively to simulate 'close prices' for random data representing the Wiener model.

500 data points are used for training and the remaining 250 points are used for evaluation. The training data points represent approximately 2 years of history prior to January 1, 2007. The training data are divided into two equal parts. In the first part, the scale parameters $a_i,\ i = 1, \ldots, p$ of the $AR - ABS(p)$ model of order p are evaluated for each parameter p. The second part of training data is used to optimize p. The evaluation data points represent approximately one year of history after January 1, 2007.

Different assets are quoted in different currencies. Thus, some numbers are large. For example, the 'kospi' data is quoted in South Korea Won. Series from all major sectors are considered: energy (brentcrudeoil), commodities (cocoa, coffee, leanhogs), currency exchange (audusd), interest rates (eurodollar), equity indexes (kospi), and bonds (tnote5y).

Table 1 shows that the best results were obtained using RW in the simulated Wiener process (as expected) and in time series of cocoa futures prices. Simulation of the Wiener process has been repeated for 10 times and $p = 1$ was always optimal. In the time series of seven other assets, the best results were provided by $AR - ABS(p)$ models of order p with the optimized order $p = p_{opt} > 1$. This indicates that in most cases asset holders are not playing equilibrium strategies, corresponding to the Wiener model. An interesting exception is futures prices of cocoa. A possible explanation is that the cocoa market is controlled by a few major players who are using buying-selling strategies close to the Nash equilibrium. The situation is different in the remaining seven markets, where the prices depend on actions of many players. Apparently most of them are looking for greater profits and are not following equilibrium strategies.

6 Web site for distance graduate studies

The stock-exchange game model is a part of the general on-line system for graduate studies and scientific collaboration [12,13].

Table 1 The residuals of next-day predictions of different instruments by different models

Asset	$AR - ABS(1)1$	$AR - ABS(p_{opt})$	p_{opt}	RW
Audusd	0.004525	0.004521	3	0.00453
Brentcrudeoil	0.9676	0.9656	17	0.97
Cocoa	22.572	22.572	1	22.572
Coffee	1.2863	1.2712	12	1.28
Eurodollar	0.049286	0.04926	19	0.0493
Kospi	2.5672	2.5536	16	2.59
Leanhogs	0.6798	0.6797	2	0.68
Random	0.7654	0.7654	1	0.765
Tnote5y	0.1987	0.1986	5	0.2

The main web site (last updated December 11, 2009) is at:

http://soften.ktu.lt/~mockus.

The mirror sites are at:

http://pilis.if.ktu.lt/~jmockus,
http://optimum2.mii.lt,
http://kopustas.elen.ktu.lt/~jmockus,
http://mockus.org/optimum.

Examples are in the form of Java applets and can be started by any browser with Java support (assuming that both Java and Javascript are enabled). Applets were compiled by java1.6, thus they may not work correctly using lesser versions of Java.

The common feature of examples is that all of them are solved using optimization techniques, including the global and discrete optimization. Several versions of SEGM are in the section: ‘Global Optimization’, subsection: ‘Stock exchange Model’, sub subsection: ‘Nash Equilibrium, examples of global optimization in a simplest stock exchange model’. The URL of the last stable version at the main server is http://soften.ktu.lt/~mockus/stockedvinas/indre.html.

To start applets in other synchronized servers the URL should be changed accordingly. Statistical simulation of SEGM is a time consuming process if I and J are large. Thus, a multiprocessor version is planed. Now the software works up to $I = J = 8$. In the multi-processor implementation, number of major players I will be defined by a number of processors. The multi-processor project is for master thesis in Informatics.

In the Internet environment, we need platform-independent languages for running software on remote computers. Java is more efficient for scientific calculations among such languages. Java applets provide a unique possibility for student-teacher interactions. Students can run the programs remotely and teachers can test students’ results on-line. The Java servlet mode is targeted at users without Java support.

This web-based software is for on-line graduate studies investigating and developing various optimization models. The web-site was started in the Lithuanian language because most students preferred it. English comments are added to clarify important examples and to help international students. All the tutorials and other general documents are in English.

The web-site is divided into several sections. The General Description section includes a description of theory and applications. General software for optimization is in the section Software Systems. The main part of general software is Global Minimizer by Java (GMJ). The GMJ software is an open framework designed for different global optimization methods, different optimization models, and the respective tools for visualization of the results. Linear Programming is represented by the open-source Java software LPJ. The tools for time series prediction by minimizing absolute deviations $AR - ABS(p)$ are also in this section. Models of continuous global optimization are in the section Global Optimization. Most of the models are economic, including optimal investment and various applications of Nash equilibrium in different problems of competition. Several versions of the stock-exchange game model are in this section as well. Models of discrete and linear programming are in Section Discrete Optimization. Examples of sequential statistical decision theory [2] are described in the same section. An example of video conferencing in English (recorded lectures) is in the section News’. The archive of entire web site optimum.tgz is in the News section as well. The remaining part of the News section contains current messages for local students, mainly in Lithuanian. The system uses two languages: English and Lithuanian. The video-conferencing is mainly in Lithuanian. The Web-based textbooks are all in English. The home work descriptions are either in English or Lithuanian, depending on the authors preferences.

7 Concluding remarks

We argue that Game Theory is a suitable framework to model financial markets because future market price of financial assets depend on predictions (and subsequent actions) of market participants with conflicting interests. For example, the simple stock exchange game model SEGM, based on the assumption that market players set their buying and selling levels for the next iteration via the autoregressive model *AR(p)* of order p selected by minimizing deviations from Nash Equilibrium (NE) can be used to explain the traditional efficient market theory as the Nash equilibrium. Simulations also explain stock market reaction to deliberately set non-NE strategies of a major stockholder, such as strategies designed to lower the value of assets.

While SEGM may be too simplistic for practical forecasting, it can serve as a useful tool for studies of market behavior by presenting an easy way of simulating different scenarios of player strategies. We have compared SEGM with eight actual financial time series and found the results to be the same, in one case, and different, but close to the Nash equilibrium in the remaining cases. A possible explanation of the difference is that, in the latter cases, major players are not following NE strategies. The similarity of results obtained using the simple SEGM model and real data indicates that the approach, based on the assumption of optimal behavior by market participants, may be suitable and useful in studies of real financial markets.

Using SRGM simulation we see that simplistic prediction of future trends is a reason of financial instability. This way SRGM helps students of business informatics to understand better financial disasters that we are witnessing.

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